

- ŠÁRKA STEJSKALOVÁ, *The indestructibility of the tree property.*

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We show that the tree property at  $\omega_2$  in the Mitchell model  $V[M]$  is indestructible by all ccc forcing notions which live in an intermediate model  $V[\text{Add}(\omega, \kappa)] \subseteq V[M]$ , provided we start with a supercompact cardinal  $\kappa$  ( $\kappa$  becomes  $\omega_2$ ). This shows that it is consistent that a large class of ccc forcings cannot add new  $\omega_2$ -Aronszajn trees (for instance, consistently, no ccc forcing living in  $L$  adds an  $\omega_2$ -Aronszajn tree). With a fancier forcing, this result extends to all forcings which are (i)  $\omega_1$ -closed and  $\omega_2$ -cc and (ii)  $\omega_2$ -directed closed. The result generalizes to cardinals larger than  $\omega_2$  and allows applications to Prikry-style forcing notions.

The work is joint with R. Honzik.